

## Higgs boson decay into $Z$ bosons and a photon

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**ABSTRACT:** We present a brief summary of our one-loop calculation of the width  $\Gamma(H \rightarrow ZZ\gamma)$  in the standard model for Higgs boson masses  $195 \text{ GeV} \leq m_H \leq 250 \text{ GeV}$ . The helicity amplitudes contain a contribution from the anomalous  $ZZ\gamma$  triangle graph and the most dominant helicity combinations for the  $Z$  bosons and the photon are when one of the  $Z$  bosons is longitudinally polarized and the other has the same helicity as the photon. This decay is, however, highly suppressed. The ratios of  $\Gamma(H \rightarrow ZZ\gamma)$  to other one-loop Higgs decays are  $\Gamma(H \rightarrow ZZ\gamma)/\Gamma(H \rightarrow \gamma\gamma) \sim \Gamma(H \rightarrow ZZ\gamma)/\Gamma(H \rightarrow \gamma Z) \lesssim 10^{-7}$ .

**KEYWORDS:** Standard Model, Higgs Physics.

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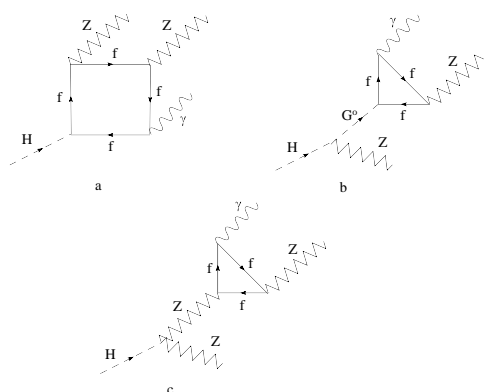
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**1. Introduction**

Like the decay  $H \rightarrow Z\gamma\gamma$  [1], the lowest order contribution to the decay  $H \rightarrow ZZ\gamma$  takes place at the one-loop level. The Feynman diagrams for these processes are similar, with triangle and box contributions. Apart from the obvious difference in the allowed Higgs mass ranges for the two processes, they happen to be sensitive to different manifestations of the axial vector anomaly. In addition, the helicity structure of the dominant decay amplitude for  $H \rightarrow ZZ\gamma$  is rather unusual in that one  $Z$  is longitudinally polarized and the other has the same polarization as the photon. As the results outlined below confirm, the observation of these unique features is problematic because the width  $\Gamma(H \rightarrow ZZ\gamma)$  is extremely small.

**2. Outline of the calculation**

The relevant Feynman diagrams are illustrated [2] in figure 1. Our choice of a nonlinear gauge is discussed in reference [1] and here, too, charge conjugation symmetry ensures that only charged fermions appear in the loops [3]. In figure 1a, the coupling of the Higgs boson



**Figure 1:** Some representative Feynman diagrams for the decay process  $H \rightarrow ZZ\gamma$  are shown. In figures 1a, 1b, and 1c, we include all charged fermions  $f$  of the third generation.

to the fermion is proportional to the fermion mass and we include only the contribution from the third generation. The Goldstone boson contribution in figure 1b is also fermion mass dependent and again we include only the third generation.

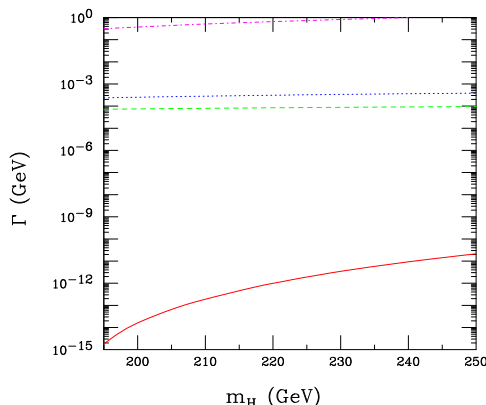
The contribution of the diagram of figure 1c, apart from the dependence on the mass of the fermion in the loop, is proportional to  $N_c^f Q_f g_A^f g_V^f$ . Here,  $N_c^f$  is the number of fermion colours ( $N_c^f = 1$  for a lepton,  $N_c^f = 3$  for a quark),  $Q_f$  is the fermion electric charge in units of the proton charge,  $g_A^f = T_3^f$  and  $g_V^f = T_3^f - 2Q_f \sin^2 \theta_W$  are the axial-vector and the vector coupling constants, respectively.  $T_3^f$  is the third component of the weak isospin, and  $\theta_W$  is the weak mixing angle. The contribution of the diagram of figure 1c consists of two parts. One depends on the mass of the charged fermion in the loop (it vanishes for a massless fermion), and the other is independent of the fermion mass. The latter gives an anomalous contribution [4]. However, it is clear that the inclusion of all charged fermions of a given generation will cancel this anomalous contribution since  $\sum_f N_c^f Q_f g_A^f g_V^f = 0$ <sup>1</sup>. Furthermore, if all of the members of a particular generation had the same mass, the total contribution of that generation would vanish. For this reason we included only the charged fermions of the third generation in the evaluation of the diagram of figure 1c. (The other two generations give negligible contributions.)

The process  $H \rightarrow ZZ\gamma$  will be dominated by the decays  $H \rightarrow \gamma\gamma$ ,  $H \rightarrow \gamma Z$ , and especially  $H \rightarrow ZZ$  which occurs at tree level. In order to facilitate the discrimination of  $H \rightarrow ZZ\gamma$  from these dominant decay modes and account for some of the possible experimental limitations, we imposed cuts on  $Z$  boson and photon kinematic variables as discussed in [1]. The cut on the three momenta is  $|\vec{p}|_{\text{cut}} = 5 \text{ GeV}$  and cut on the opening angles is  $\theta_{\text{cut}} = \pi/24$ . These kinematic cuts facilitate the experimental tagging of the  $Z$  bosons and photon. They provide minimum opening angles between the  $Z$  bosons and the photon, exclude contributions of the back-to-back  $Z$  bosons and photons, exclude soft photons, and also improve the numerical stability of the calculations. Using the calculational approach described in [1], we computed the decay width, its distributions with respect to the invariant mass and photon energy, and checked the gauge invariance of the result by replacing the photon polarization vector with its momentum.

The result of the calculation for the decay width  $\Gamma(H \rightarrow ZZ\gamma)$  as function of the Higgs boson mass  $m_H$  is shown in figure 2. For comparison, in this figure we also included the decay widths  $\Gamma(H \rightarrow ZZ)$ ,  $\Gamma(H \rightarrow \gamma\gamma)$ , and  $\Gamma(H \rightarrow \gamma Z)$  [5, 6]. It is clear from this figure that the decay width  $\Gamma(H \rightarrow ZZ\gamma)$  is several orders of magnitude smaller than those of  $H \rightarrow ZZ$ ,  $H \rightarrow \gamma\gamma$ , and  $H \rightarrow \gamma Z$ . For Higgs boson masses  $195 \text{ GeV} \leq m_H \leq 250 \text{ GeV}$ , the ratios of the decay widths are  $\Gamma(H \rightarrow ZZ\gamma)/\Gamma(H \rightarrow \gamma\gamma) \sim \Gamma(H \rightarrow ZZ\gamma)/\Gamma(H \rightarrow \gamma Z) \lesssim 10^{-7}$  and  $\Gamma(H \rightarrow ZZ\gamma)/\Gamma(H \rightarrow ZZ) \lesssim 10^{-10}$ . To identify the origins of the smallness of these ratios, we note that, in addition to the cuts that we imposed on the decay products of  $H \rightarrow ZZ\gamma$ , which decrease the value of  $\Gamma(H \rightarrow ZZ\gamma)$ , there is also the suppression from three-body phase space, and from the higher order in the coupling constant  $\alpha$ . These, however, do not completely account for the suppression. There are other differences in the

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<sup>1</sup>For the decay  $H \rightarrow \gamma\gamma Z$  (calculated in [1]), the corresponding diagram in figure 1c gives an anomalous contribution that is proportional to  $N_c^f Q_f^2 g_A^f$ , and when all charged fermions of a given generation are included in the loop, this anomaly will also vanish,  $\sum_f N_c^f Q_f^2 g_A^f = 0$ .



**Figure 2:** The decay widths as function of  $m_H$  for several decay modes of the Higgs boson are shown. The solid line is  $\Gamma(H \rightarrow ZZ\gamma)$ , the dashed line is  $\Gamma(H \rightarrow \gamma\gamma)$ , the dotted line is  $\Gamma(H \rightarrow \gamma Z)$ , and the dotdashed line is  $\Gamma(H \rightarrow ZZ)$ . The cuts imposed on  $\Gamma(H \rightarrow ZZ\gamma)$  are  $|\vec{p}'|_{\text{cut}} = 5 \text{ GeV}$  and  $\theta_{\text{cut}} = \pi/24$ .

various decay amplitudes, which contribute to the small ratios. For instance, in the case of  $H \rightarrow \gamma\gamma$ , the decay amplitude receives contributions from charged fermion loops as well as a substantial contribution from  $W$  boson loops, whereas in the decay  $H \rightarrow ZZ\gamma$ , there are no  $W$  boson loop contributions and the inclusion of anomalous triangle diagram, figure 1c, further suppresses the amplitude. As a result of these differences, the simple power counting method for estimating the size of the ratio of the decay widths  $\Gamma(H \rightarrow ZZ\gamma)/\Gamma(H \rightarrow \gamma\gamma)$  is rather unreliable.

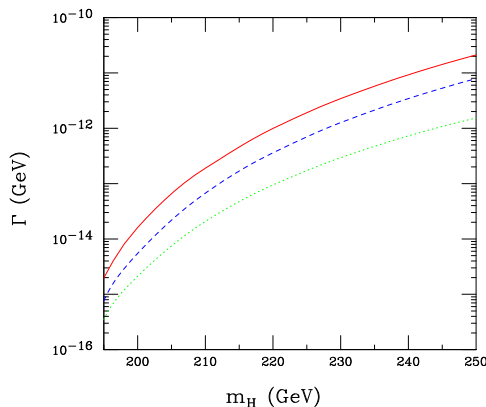
To investigate the dependence of the decay width  $\Gamma(H \rightarrow ZZ\gamma)$  on the helicities of the produced  $Z$  bosons and the photon, we can use Bose symmetry and the  $CP$  invariance to obtain relations among the helicity amplitudes  $\mathcal{A}_{\lambda\lambda'\lambda_\gamma}$ . Here,  $\lambda$  and  $\lambda'$  are the helicities of the  $Z$  bosons and  $\lambda_\gamma$  is the helicity of the photon. As a consequence of these symmetries, the decay width  $\Gamma_{\lambda\lambda'\lambda_\gamma}$  satisfies the following relations

$$\Gamma_{\lambda\lambda'\lambda_\gamma} = \Gamma_{\lambda'\lambda\lambda_\gamma}, \tag{2.1}$$

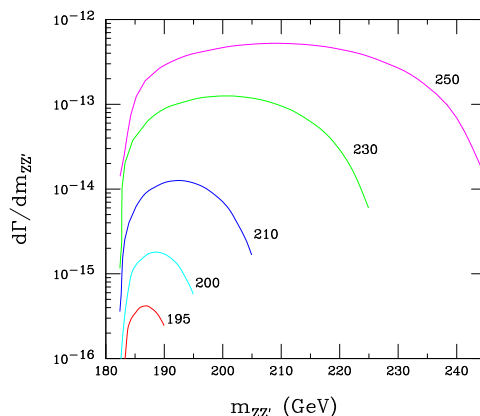
$$\Gamma_{\lambda\lambda'\lambda_\gamma} = \Gamma_{-\lambda-\lambda'-\lambda_\gamma}. \tag{2.2}$$

In figure 3, we show the result of our calculation of the decay width  $\Gamma(H \rightarrow ZZ\gamma)$  as function of the Higgs boson mass  $m_H$ , for different helicities of the  $Z$  bosons and the photon. As it is clear from this figure, the decay widths  $\Gamma_{0++} = \Gamma_{0--}$ , which correspond to the case when one of the  $Z$  bosons is longitudinally polarized and the other  $Z$  boson has the same helicity as that of the photon, are the most dominant. This dominance is stronger for the higher Higgs boson masses. This pattern of the polarization states may be viewed as a signature for the decay products of the process  $H \rightarrow ZZ\gamma$ , since a longitudinal-transverse helicity combination cannot occur for the  $Z$  pair from  $H \rightarrow ZZ$ .

In figure 4, we show the invariant mass distribution  $d\Gamma(H \rightarrow ZZ\gamma)/dm_{ZZ'}$  as function of the  $Z$  boson pair invariant mass  $m_{ZZ'}$ , and in figure 5, we show the energy distribution  $d\Gamma(H \rightarrow ZZ\gamma)/dE_\gamma$  as function of the photon energy  $E_\gamma$ .



**Figure 3:** The decay widths  $\Gamma(H \rightarrow ZZ\gamma)$  as function of  $m_H$  for different helicities  $(\lambda\lambda'\lambda_\gamma)$  of the  $Z$  bosons and the photon are shown. The solid line is for the unpolarized case, the dashed line is for  $(0++)$  and the dotted line is for  $(0+-)$ . The helicity combinations  $(00+) = (00-)$ ,  $(+++)$   $=$   $(---)$ ,  $(++-)$   $=$   $(--+)$ , and  $(+-+)$   $=$   $(-+-)$   $=$   $(-++)$   $=$   $(+--)$  that are not shown are negligible.

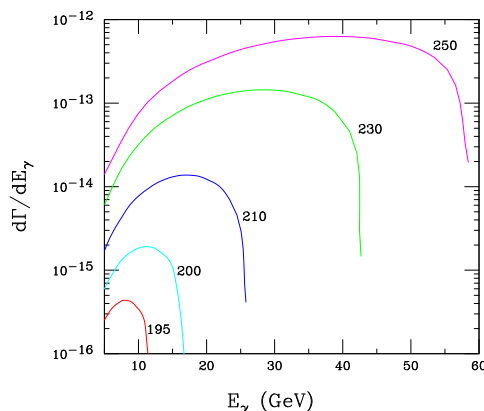


**Figure 4:** The invariant mass distributions  $d\Gamma(H \rightarrow ZZ\gamma)/dm_{ZZ'}$  as function of  $m_{ZZ'}$ , the invariant mass of the final  $Z$  bosons, for Higgs masses of  $m_H = 195, 200, 210, 230,$  and  $250$  GeV are shown.

### 3. Summary and conclusions

In the standard model, the three-body decay of the Higgs boson  $H \rightarrow ZZ\gamma$  is highly suppressed. However, this decay mode has some interesting features that separate it from other one-loop decay modes such as  $H \rightarrow \gamma\gamma$  and  $H \rightarrow \gamma Z$ . One is the absence of  $W$  boson contributions in any of the loops in the Feynman diagrams for the  $H \rightarrow ZZ\gamma$ . Its amplitudes are dominated by top quark loops and therefore sensitive to top- $Z$  couplings. Also, there is the presence of an anomalous vertex in the  $s$ -channel  $Z$  exchange diagram of figure 1c, which might be studied were it not for the smallness of the decay width.

Our explicit calculations show that the most dominant helicity combinations for  $H \rightarrow ZZ\gamma$  occur when one of the  $Z$  bosons is longitudinally polarized and the other  $Z$  boson



**Figure 5:** The energy distributions  $d\Gamma(H \rightarrow ZZ\gamma)/dE_\gamma$ , as function of the photon energy  $E_\gamma$ , for the Higgs masses of figure 4 are shown.

and the photon have the same helicity. This is a result that was not apparent at the outset. With enough statistics, this feature might be used to discriminate the  $Z$  pairs of  $H \rightarrow ZZ\gamma$  from those of  $H \rightarrow ZZ$ , since the helicities of the  $Z$  pair in the latter decay cannot be in the combination longitudinal-transverse.

In summary, we find that the decay width for the process  $H \rightarrow ZZ\gamma$  is exceedingly small compared to those of  $H \rightarrow \gamma\gamma$  and  $H \rightarrow \gamma Z$ , and that the suppression is greater than the simple phase space and coupling constant accounting might suggest. Therefore, we expect not to detect a signal of this mode at the standard model level. While it is conceivable that some non-standard model interaction could enhance this decay, it is by no means clear that this could occur without at the same time enhancing  $H \rightarrow \gamma\gamma$  and  $H \rightarrow \gamma Z$ .

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